Fajgelbaum and Schaal (2020): “Optimal Transport Networks in Spatial Equilibrium”

- Large investments in infrastructure
  - 20% of World Bank spending, 6% of government spending around the world
  - How should these investments be allocated in a transport network?

- Challenges
  - Investments in one segment affect returns in others
  - Reallocation of economic activity and trading routes
  - Dimensionality
This Paper

- Develop a framework to study optimal transport networks in general equilibrium

1. Solve global optimization over the space of networks
   - given any primitive fundamentals
   - in a neoclassical trade framework (with labor mobility)

2. Apply to road networks in 24 European countries
   - how large are the gains from expansion and the losses from misallocation of current networks?
   - what are the regional effects?
Key Features

- **Neoclassical Trade Model on a Graph**
  - Infrastructure impacts shipping cost in each link

- **Sub-Problems:**
  - how to ship goods through the network? ("Optimal Flows")
  - how to build infrastructure? ("Optimal Network")

- **Optimal flows/routes**
  - Numerically very tractable
  - Especially using dual approach (convex optimization in space of prices)

- **Full problem (Flows+Network+GE) inherits numerical tractability**
  - Infrastructure investment expressed as function of equilibrium prices
  - Requires general problem to be convex (congestion in transport)
Model
Preferences and Technologies

- \( J = \{1, \ldots, J\} \) locations
  - \( N \) traded goods aggregated into \( c_j \)
  - 1 non-traded good \( h_j \) in fixed supply (can make it variable)
  - \( L_j \) workers located in \( j \) (fixed or mobile)

- Homothetic and concave utility in \( j \),
  \[
  U(c_j, h_j)
  \]
  where
  \[
  c_j L_j = D_j \left(D_1^j, \ldots, D_N^j\right)
  \]
  - \( D_j(\cdot) \) homogeneous of degree 1 and concave (e.g., CES)

- Output of sector \( n \) in location \( j \) is:
  \[
  Y^n_j = F^n_j \left(L^n_j, V^n_j, X^n_j\right)
  \]
  - \( F^n_j(\cdot) \) is neoclassical
  - \( V^n_j, X^n_j \) = other primary factors and intermediate inputs

- Special cases
  - Ricardian model, Armington Specific-factors, Heckscher-Ohlin, Endowment economy, Rosen-Roback...
Underlying Graph

- The locations are arranged on an \textit{undirected} graph
  - $\mathcal{J} = \{1, \ldots, J\}$ nodes
  - $\mathcal{E}$ edges

- Each location $j$ has a set $\mathcal{N}(j)$ of \textquotedblleft neighbors\textquotedblright{} (directly connected)
  - Shipments flow through neighbors
  - \textquotedblleft Neighbors\textquotedblright{} may be geographically distant
    - Fully connected case $\mathcal{N}(j) = \mathcal{J}$ is nested

- E.g.: Spain, $\sim 50 \text{ km} \times 50 \text{ km}$ square network, \#$\mathcal{N}(j) = 8$
Transport Technology

- Per-unit cost $\tau_{jk}^n (Q_{jk}^n, I_{jk})$ denominated in units of good itself (iceberg)
  - $Q_{jk}^n =$ quantity of commodity $n$ from $j$ to $k \in \mathcal{N}(j)$
  - $l_{jk} =$ index of road quality/capacity (number of lanes, highway, ...)

- **Decreasing returns to transport:** $\frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} \geq 0$
  - “Congestion” in short, may account for travel times, road damage, fixed factors in transport technologies...
  - Alternatively, cross-good congestion: $\tau_{jk}^n (\sum_n Q_{jk}^n, I_{jk})$ denominated in units of the bundle of traded goods

- **Positive returns to infrastructure:** $\frac{\partial \tau_{jk}^n}{\partial l_{jk}} < 0$
  - Infrastructure investment lower trade costs (# of lanes, whether road is paved, etc.)
  - The transport network chosen by planner is defined by $\{l_{jk}\}_{j \in \mathcal{J}, k \in \mathcal{N}(j)}$
Network Technology

Constraint on flows:

\[ D_j^n + \sum_{n'} X_{jn'} + \sum_{k \in \mathcal{N}(j)} (1 + \tau_{jk}^n) Q_{jk}^n \leq Y_j^n + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n \]

Consumption + Intermediate Use + Exports \hspace{2cm} Production + Imports

Building infrastructure \( l_{jk} \) takes up \( \delta_{jk}^l l_{jk} \) units of a scarce resource in fixed supply \( K \) ("asphalt")

- Building cost \( \delta_{jk}^l \) may vary across links
- e.g. due to ruggedness, distance...

Aggregate resource constraint:

\[ \sum_j \sum_{k \in \mathcal{N}(j)} \delta_{jk}^l l_{jk} \leq K \]
Planner’s Problem Given Network

**Definition**

The planner’s problem given the infrastructure network is

\[ W_0 \left( \{ l_{jk} \} \right) = \max_{c_j, L_j, V_j^n, X_j^n, L_j, Q_j^n} \max_u \]

subject to (i) availability of traded and non-traded goods,

\[ c_j L_j \leq D_j (D_j) \text{ and } h_j L_j \leq H_j \text{ for all } j; \]

(ii) the balanced-flows constraint,

\[ D_j^n + \sum_{n'} X_j^n n' + \sum_{k \in N(j)} \left( 1 + \tau^k_{jk} \left( Q_j^n, l_{jk} \right) \right) Q_j^n = Y_j^n + \sum_{i \in N(j)} Q_{ij}^n \text{ for all } j, n; \]

(iii) free labor mobility,

\[ L_j u \leq L_j U (c_j, h_j) \text{ for all } j; \]

(iv) local and aggregate labor-market clearing; and

(v) factor market clearing and non-negativity constraints.
Optimal Flows Problem

- Multipliers $P^n_j$ of balanced-flow constraints = price of $n$ in $j$ in decentralization

- No-arbitrage condition:

$$P^n_k \leq P^n_j \left( 1 + \tau^n_{jk} + \frac{\partial \tau^n_{jk}}{\partial Q^n_{jk}} Q^n_{jk} \right), = \text{ if } Q^n_{jk} > 0$$

  - Key property: $Q^n_{jk}$ invertible from $P^n_j$, $P^n_k$

- Dual solution coincides with primal

  - Dual: convex optimization with linear constraints in smaller space (of prices):

$$\inf_P \mathcal{L} (C(P), L(P), Q(P); P)$$

  - Use FOCs and substitute for $C, Q,...$, as function of $P$, then minimize over Lagrange multipliers
  - Convex minimization problem in fewer variables with just non-negativity constraints (just $P$)

  - Efficient algorithms are guaranteed to converge to global optimum (Bertsekas, 1998)
Optimization over Transport Network

Definition

The full planner’s problem with labor mobility is

\[ W = \max_{\{l_{jk}\}} \{ W_0 (\{l_{jk}\}) \} \]

subject to: (a) the building constraint,

\[ \sum_j \sum_{k \in N(j)} \delta^{l}_{jk} l_{jk} = K; \]

and (b) the bounds

\[ l_{jk} \leq l_{jk} \leq \bar{l}_{jk} \]

At the global optimum, the optimal network satisfies

\[ P_K \delta^{l}_{jk} \geq \sum_{n} P^n_j Q^n_{jk} \times \left( -\frac{\partial T^n_{jk}}{\partial l_{jk}} \right), \quad \text{if } l_{jk} > l_{jk} \]

Reduces optimization to space of prices \(\rightarrow\) Full problem inherits tractability of optimal flows

Proposition

If the function \(Q_{\tau_{jk}} (Q, l)\) is convex in \(Q\) and \(l\), the full planner’s problem with mobile labor (immobile labor) is a quasiconvex (convex) optimization problem.
Log-Linear Transport Technology

- Log-linear transport technology:
  \[ \tau_{jk}(Q, I) = \delta_{jk} \frac{Q^\beta}{I^\gamma} \]

  - Global convexity if \( \beta > \gamma \)

- Optimal network

  \[ I_{jk}^* \propto \left[ \frac{1}{\delta_{jk}^I \left( \delta_{jk}^T \right)^{\frac{1}{\beta}}} \left( \sum_{n:P_k^n > P_j^n} P_j^n \left( \frac{P_k^n}{P_j^n} - 1 \right)^{\frac{1+\beta}{\beta}} \right) \right]^{\frac{\beta}{\beta-\gamma}} \]
Decentralization

Atomistic traders into shipping each good $n$ from $o$ to $d$ solve:

$$\pi_{od}^n = \max_{r=(j_0, \ldots, j_\rho) \in R_{od}} p_d^n - p_o^n - \sum_{k=0}^{\rho-1} p_{jk}^n \tau_{jkj_{k+1}}^n - \sum_{k=0}^{\rho-1} p_{jk}^n t_{jkj_{k+1}}^n$$

Sourcing Costs

Transport costs

Taxes

Proposition

(Welfare Theorems) If the tax on shipments of product $n$ from $j$ to $k$ is

$$t_{jk}^n = \frac{\partial \log \tau_{jk}^n}{\partial \log Q_{jk}^n} \tau_{jk}^n,$$

then the competitive allocation coincides with the planner’s problem.

Builders choose infrastructure, receive a per-unit toll $\theta_{jk}^n$, and take prices as given:

$$\max_{l_{jk}} \sum_n \theta_{jk}^n Q_{jk}^n \left( l_{jk}; p_k^n, p_j^n \right) - p_K \delta_{jk}^l l_{jk}$$

consistent with ZP

Proposition

(Network Decentralization) If the toll $\theta_{jk}^n$ is consistent with the optimal Pigouvian tax, then the decentralized infrastructure implements the optimal network investment.
Examples
One Good in a Square
One Traded Good, Endowment Economy, Output 10x Larger at Center, Uniform Fixed Population

(a) Population
(b) Productivity
One Good in a Square
Optimal Network, $K = 1$
One Good in a Square
Optimal Network, $K = 100$
Role of Building Costs
20 random cities across uniform geography, convex case

Building Cost: $\delta_{jk}^I = \delta_0 \text{Distance}_{jk}^{\delta_1}$
Role of Building Costs

Adding a mountain, a river, bridges, and water transport

Building Cost: \( \delta_{jk} = \delta_0 \text{Distance}_{jk}^{\delta_1} \left(1 + |\Delta \text{Elevation}_{jk}|^{\delta_2}\right) \delta_3^{\text{CrossingRiver}_{jk}} \delta_4^{\text{AlongRiver}_{jk}} \)
Application
Application

- Compute optimal road network and efficiency gains across European countries
- In 24 European countries we observe
  - Road networks with features of each segment: number of lanes and national/local road (EuroGeographics)
  - Value Added (G-Econ 4.0)
  - Population (GPW)
Underlying Graph and Observed Infrastructure: Spain

- Construct the graph \((\mathcal{J}, \mathcal{E})\) with quality of actual road network for each country
  - \(\mathcal{J}\): population centroids of 0.5 x 0.5 degree (\(\sim\)50 km) cells
  - \(\mathcal{E}\): all links among contiguous cells (8 neighbors per node)
  - \(I_{jk}^{\text{obs}}\): observed infrastructure between all connected \(jk \in \mathcal{E}\)

Actual Road Network | Discretized Road Network
Calibration

- **Production technologies**: \( Y_j^n = z_j^n L_j^n \)

- **Preferences**: \( U(c, h) = c^\alpha h^{1-\alpha} \)
  
  - \( N \in \{5, 10, 15\} \) tradeable sectors with CES demand (\( \sigma = 5 \))

- **Fundamentals**: \( \{z_j, H_j\} \) such that \( \{GDP_j^{obs}, L_j^{obs}\} \) is the model’s outcome given \( \{I_{jk}^{obs}\} \)

- **Building costs** \( \delta_{jk}^I \)
  
  - Use estimates from Collier et al. (2016) → Set \( \delta_{jk}^{I,GEO} \) as function of distance and ruggedness
    
    \[
    \ln \left( \frac{\delta_{jk}^{I,GEO}}{\text{dist}_{jk}} \right) = \ln (\delta_0^I) - 0.11 \times (\text{dist}_{jk} > 50 km) + 0.12 \times \ln (\text{rugged}_{jk})
    \]
  
  - Alternative: assume that observed road networks are optimal → Back out \( \delta_{jk}^{I,FOC} \) from FOC’s using \( I_{jk}^{obs} \)
Transport Costs

- Transport technology with cross-goods congestion:

\[ \tau_{jk}^n = \delta_{jk}^{\tau} \left( \sum_n Q_{jk}^n \right)^{\beta} \]

- Choose scale to match intra-regional share of intra-national trade in Spain

- Consistent with assuming:
  - trade costs (\( \tau \)) are a linear function of shipping time
  - speed (\( S \)) is a log-linear function of the vehicles (\( V \)) and road lane kilometers (\( I \))
  - vehicles (\( V \)) is a linear function of the shipments

- Implies

\[ \ln S_{jk} = \left( \frac{\gamma}{1 + \beta} \right) \ln I_{jk} - \left( \frac{\beta}{1 + \beta} \right) \ln \left( \frac{V_{jk} \cdot \text{dist}_{jk}}{S_{jk}} \right) + \varepsilon_{jk} \]

- Use estimates from Couture, Duranton, and Turner (2018)
- Estimates imply \( \gamma = 0.10 \) and \( \beta = 0.13 \)
Model Implied Trade Flows across Spanish NUTS
Optimal Expansion and Reallocation

- **Optimal expansion**
  - Increase $K$ by 50% relative to calibration in every country
  - Build on top of existing network ($I_{jk} \geq I_{jk}^{obs}$)
  - Using both $\delta_{jk}^{I, FOC}$ and $\delta_{jk}^{I, GEO}$

- **Optimal reallocation**
  - $K$ is equal to calibrated model
  - Build anywhere ($I_{jk} \geq 0$)
  - Using $\delta_{jk}^{I, GEO}$
## Aggregate Effects (Cross-Country Average)

<table>
<thead>
<tr>
<th>Returns to Scale:</th>
<th>Benchmark</th>
<th>Non-Convex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor:</td>
<td>Fixed</td>
<td>Mobile</td>
</tr>
<tr>
<td>Optimal Reallocation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta = \delta^l,GEO )</td>
<td>1.7%</td>
<td>1.8%</td>
</tr>
<tr>
<td>Optimal 50% Expansion</td>
<td></td>
<td></td>
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<tr>
<td>( \delta = \delta^l,GEO )</td>
<td>1.7%</td>
<td>1.8%</td>
</tr>
<tr>
<td>( \delta = \delta^l,FOC )</td>
<td>0.3%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>
Regional Effects (Spain)
Optimal Expansion
Regional Effects (Spain)
Optimal Reallocation
## Optimal Placement and Regional Effects

### Panel A: Dependent Variable: Infrastructure Growth

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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</thead>
<tbody>
<tr>
<td>Reallocation</td>
<td>0.343**</td>
<td>0.125***</td>
<td>0.002</td>
</tr>
<tr>
<td>Expansion (GEO)</td>
<td>0.151</td>
<td>0.071</td>
<td>0.007*</td>
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<tr>
<td>Expansion (FOC)</td>
<td>-0.418***</td>
<td>-0.235***</td>
<td>-0.010</td>
</tr>
<tr>
<td>Observations</td>
<td>868</td>
<td>868</td>
<td>868</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.29</td>
<td>0.24</td>
<td>0.04</td>
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### Panel B: Dependent Variable: Population Growth

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<tbody>
<tr>
<td>Reallocation</td>
<td>-0.001</td>
<td>-0.000</td>
<td>-0.000</td>
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<tr>
<td>Expansion (GEO)</td>
<td>0.008</td>
<td>0.008</td>
<td>0.001</td>
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<tr>
<td>Expansion (FOC)</td>
<td>-0.061***</td>
<td>-0.060***</td>
<td>-0.008***</td>
</tr>
<tr>
<td>Observations</td>
<td>868</td>
<td>868</td>
<td>868</td>
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<tr>
<td>Adjusted R-squared</td>
<td>0.53</td>
<td>0.54</td>
<td>0.80</td>
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European Countries

Actual Road Network

Discretized Road Network
Optimal vs. Planned Investments

Figure: Optimal Expansion

Figure: TEN-T Network
Conclusion

- We develop and implement a framework to study optimal transport networks
  - Neoclassical model (with labor mobility) on a graph
  - Optimal Transport with congestion
  - Optimal Network

- Application to road networks in Europe

- Other potential applications
  - Political economy / competing planners
  - Model-based instruments for empirical work on impact of infrastructure
  - Optimal investments in developing countries
  - Optimal transport of workers
  - Absent forces: agglomeration, dynamics